**MAT 140 Lab**

(D) Interactive Geometry

1. Geometric constructions

Prepared by Younhee Lee (March 2021)

**Construction of Parabola in Geometer’s SketchPad**

Source of this Activity: Exploring Conic Sections with The Geometer’s SketchPad © 2002 Key Curriculum Press

**Constructing a Sketchpad Model**

1. Diagram

   Description automatically generatedOpen a new sketch. Use the Line tool to draw a horizontal line near the bottom of the screen. This line represents the bottom edge of the paper.
2. Draw a point *A* above the line, roughly centered between the left and right edges of the screen.
3. Construct a point *B* on the horizontal line.
4. Construct the “crease” formed when point *B* is folded onto point *A*.
5. Drag point *B* along its line. If you constructed your crease line correctly, it should adjust to the new locations of point *B.*
6. Select the crease line and choose **Trace Line** from the Display menu.
7. Drag point *B* along the horizontal line to create a collection of crease lines.
8. Drag point *A* to a different location, then, if necessary, choose **Erase Traces** from the Display menu.
9. Drag point *B* to create another collection of crease lines.

Retracing creases for each location of point A is certainly faster than folding paper. But we can do better. Ideally, your crease lines should relocate automatically as you drag point *A*. Sketchpad’s powerful Locus command makes this possible.

1. Turn tracing off for your original crease line by selecting it and once again choosing **Trace Line** from the Display menu.
2. Now select your crease line and point *B*. Choose **Locus** from the Construct menu. An entire set of creases will appear: the locus of crease locations as point *B* moves along its path. If you drag point *A*, you’ll see that the crease lines readjust automatically.

**Questions**

Q1: How does the appearance of the curve change as you move point *A* closer to the horizontal line?

Q2: How does the appearance of the curve change as you move point *A* away from the horizontal line?

**Playing Detective**

Each crease line on your paper touches the parabola at exactly one point. Another way of saying this is that each crease is tangent to the parabola. By engaging in some detective work, you can locate these tangency points and use them to construct just the parabola without its creases.

Diagram, shape

Description automatically generated

1. The exact point of tangency lies at the intersection of two lines—the crease line and another line not shown here. Construct this line in your sketch as well as the point of tangency, point D.
2. Select point *D* and point *B* and choose **Locus** from the Construct menu. If you’ve constructed point *D* correctly, you should see a curve appear precisely in the white space bordered by the creases.

**How to Prove It**

The construction above seems to generate parabolas. Can you prove that it does? Try developing a proof on your own or work through the following steps and questions.

The picture above should resemble your Sketchpad construction. Line *EF* (the perpendicular bisector of segment *AB*) represents the crease line formed when point *B* is folded onto point *A.* Point *D* sits on the curve itself.

**Questions**

Q3: Assuming point *D* traces a parabola, which two segments must you prove equal in length?

Q4: Use a triangle congruence theorem to prove that .

Q5: Use the distance definition of a parabola and the result from Q4 to prove that point *D* traces a parabola. [Remember, a parabola is the set of points equidistant from a fixed point (the focus) and a fixed line (the directrix).]

**Notes for Teachers**

Prerequisites: For students to complete the proof, they’ll need to know the distance definition of a parabola and the SAS triangle congruency theorem.

Activity Time: 50–60 minutes.

Answers:

Q1: As point A approaches the horizontal line, the curve appears “narrower.”

Q2: As point A moves away from the horizontal line, the curve appears “wider.”

Q3: You must prove that AD = DB.

Q4: Since the crease line is the perpendicular bisector of segment AB, we have CA = CB and And, of course, CD = CD. Thus, by the SAS triangle congruency theorem, .

Q5: Since , corresponding sides AD and DB are equal in length.

In step 12, students are asked to construct the point of tangency to their parabola. To do so, construct a line k through point B perpendicular to the horizontal line. The point of tangency lies at the intersection of line k with the crease line.

It’s interesting to consider whether a parabola has asymptotes; that is, if there are lines the curve approaches but never crosses. By observing the tangent line as point B moves farther and farther away from the focus, we see that the curve becomes more and more perpendicular to the directrix. So if there are asymptotes, they must be perpendicular to the directrix.

But given any perpendicular, there is a point on it that is equidistant from the focus and the directrix. Thus the curve crosses the line, and the line cannot be an asymptote.